Mathematics Education in the Anthropocene

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Abstract
Mathematics education as a field has had little interaction with issues of sustainability, yet the world faces unprecedented global and societal challenges. Human intervention has led some academics to suggest we have now entered a new era of geological time, the Anthropocene (Crutzen and Stoermer, 2000). The label 'Anthropocene', for some, signals the shift from hopes of 'saving nature' and 'solving' problems, to living with crises and problems as our new and permanent condition (Purdy, 2015). This paper asks what potential societal crises might mean for mathematics teaching and learning and offers a series of questions thrown up by considering mathematics education in the Anthropocene and some partial, fragmentary responses. One common element in these responses is a sense of returning attention to the present moment of interaction between students and teachers of mathematics.

Key Words: Anthropocene; sustainability; mathematics teaching; mathematics learning; mathematics teacher education; uncertainty; paradox; aesthetic.

1. Introduction

The world appears to face unprecedented global and societal challenges. The extent and range of potentially imminent crises threatens the sustainability of human and many other forms of life on the plant. The aim of this article is to consider what recognition of such a context could mean for mathematics teaching and learning. The first section is a review of past work linking mathematics education to issues of sustainability, a strand of thinking that goes back several decades. The more recent concept of the Anthropocene is then introduced, along with the post-human perspective it suggests. From this perspective, uncertainty, paradox and the aesthetic are among the values offered to inform a series of question and partial answers related to the teaching and learning of mathematics.

2. Mathematics education and sustainability

On the electronic repository, JSTOR (www.jstor.org) there is a single search result with all three words 'mathematics', 'education' and 'sustainability' in the title or abstract, and this result is not within the field of mathematics education. The word 'sustainability' throws up no matches, linked to environmental concerns, in electronic searches of the archives of the journals: Educational Studies in Mathematics; Research in Mathematics Education; Journal of Mathematical Behavior, to take just three. These results are indicative that there has to date not been widespread engagement with issues of sustainability within published writing on mathematics education, or at least not under that name. However, there has been considerable thinking about the place and role of mathematics in the world and a wider search of journals and books has thrown up four strands of work that could be seen as related to sustainability, which are reviewed briefly here.

In the 1980s and 1990s, Skovsmose introduced the idea of a critical mathematics education (e.g., 1994) which entailed a classification of types of knowing into: mathematical knowing (formal symbol system rules and techniques); technological knowing (applying mathematics to solve real world problems, including mathematical modelling); and, reflective knowing (including awareness of the purposes of modelling and consideration of the 'formatting power' of mathematics). The 'formatting power of mathematics' is, in part, the insight that as well as describing and representing our world, mathematics changes the world. It seems likely that political interference in the mathematical modelling of fish stocks in Canada contributed to the collapse of what was once the largest group of cod in the world (see Coles, 2013). Another example might be the way computer modelling of financial markets can create 'flash crashes' of the stock market (e.g., Wall Street 6th May 2010) when the models become self-referential and lead to irrational outcomes with real consequences. In these and countless other cases, 'mathematics itself becomes part of reality and inseparable from other aspects of society' (Skovsmose, 2001, p. 11).

Drawing on some similar roots, a second strand of thinking around sustainability is linked to the much broader tradition of mathematical modelling, within mathematics education research. Kaiser and Siriman (2006) identify 'socio-critical modelling' and an 'emancipatory perspective' (p.304) as one of six approaches to modelling, in their global survey. The emancipatory perspective on modelling is linked to ethno-mathematics (D'Ambrosio, 1999) and, through D'Ambrosio, to the work of Friere (1970) who proposed education as a mechanism by which oppressed people could come to view critically their lived reality and, as a consequence, work towards a transformation of society. For Friere, changes in society emanate from a shift in the perception of individuals regarding their situation. Education typically serves to maintain relationships of oppressor-oppressed and is inherently exploitative. The raising of class-consciousness needed in the process of liberation has reflection at its core; reflection linked strongly to action, both political and pedagogical. Skovsmose's (1994) notion of reflective knowing also has strong echoes to Friere.

More recently, two further strands of thinking have emerged in relation to sustainability. Renert (2011) has suggested the need for a 'sustainable mathematics education' and proposed a set of three stages or levels of approach: accommodation (sustainability issues are used as a context for teaching mathematical skills, e.g., statistical skills are taught using data relevant to sustainability but the meaning or implications of the data are not discussed); reformation (in which some critical thinking and discussion of values are included as a valid aspect of learning mathematics); transformation (teaching and learning mathematics becomes subordinate to a process of students becoming engaged and critical citizens, able to critique the status quo and participate in social action). A transformational approach has clear connections to Skovsmose's reflective knowing and, indeed, in a response to Renert, Gellert (2011) suggested the importance for this proposed new movement to make explicit connections with the work of critical mathematics education (Skovsmose, 1994).

The final strand identified in this review is linked to the work of Barwell (2013b) who has engaged specifically in the issue of climate change and mathematics education. Barwell engages in study of climate change both within the perspective of critical mathematics education and from a discursive psychology stance. A discursive approach echoes the work of Barbosa (2006), within the emancipatory perspective of mathematical modelling. In Barbosa's work, discussions amongst students that went beyond the pure mathematics of the model being developed were central to the process supporting students to become critically engaged citizens.

The work reviewed here is of high significance in the current moment and each strand of thinking offers an important perspective and an impetus to action. Since the 1970s, 80s and 90s our awareness of the scope and scale of global issues has shifted and there is, in 2016, perhaps a renewed urgency around issues of sustainability. The first three strands, above, share an emancipatory agenda and there are common links to the work of Friere (1970). Barwell's discursive approach represents a departure in the sense of moving away from a focus on desired or ideal methods of classroom organisation, something linked to the thinking that is explored in the next section.

3. The Anthropocene

That the world faces unprecedented global and societal challenges caused by human intervention has led some academics to suggest we have now entered a new era of geological time, the Anthropocene (Finney, 2014). The label 'Anthropocene' was proposed by Crutzen (Crutzen and Stoermer, 2000) and is taken up, for example, by the International Geosphere-Biosphere Programme (IGBP, 2004). In the IGBP (2004) publication there are two figures, now quite widely reported, that point to the accelerating impact of human activity on the planet over the last two hundred and fifty years (Figure 1) and secondly, that record the impacts of these accelerating human actions over the same timescale (Figure 2). It is the manner in which human activity has provoked change at a planetary level, particularly evident in the last fifty years, that has led to the suggestion we have entered a new geological era.

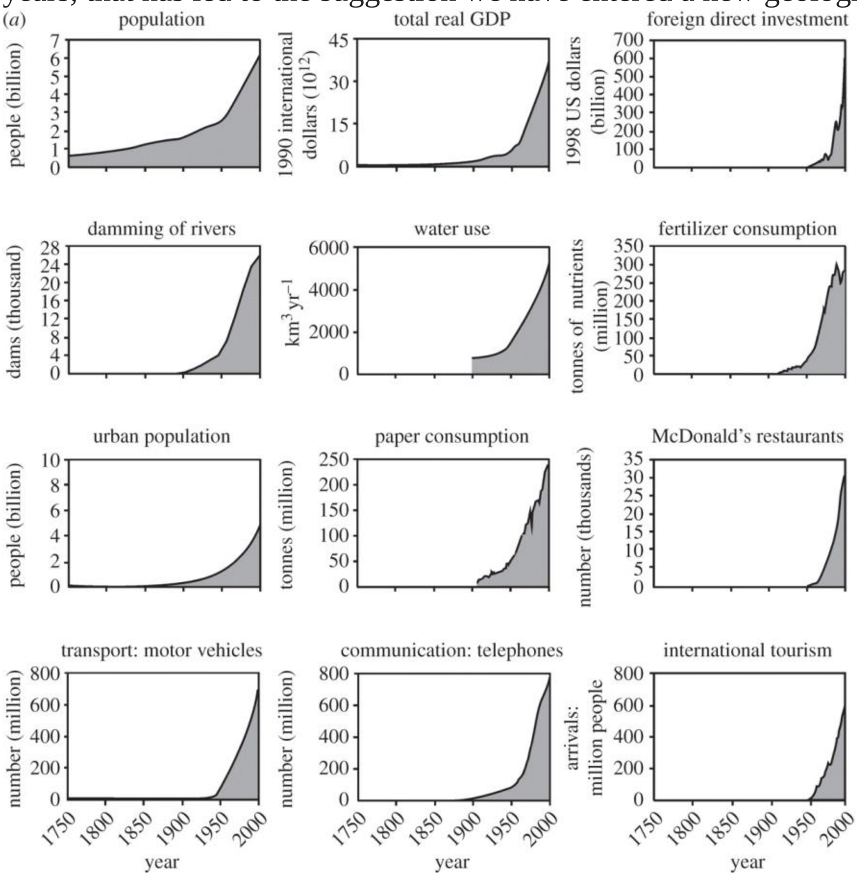

Figure 1: Evidence of accelerating human activity (IGBP, 2004, p.132)

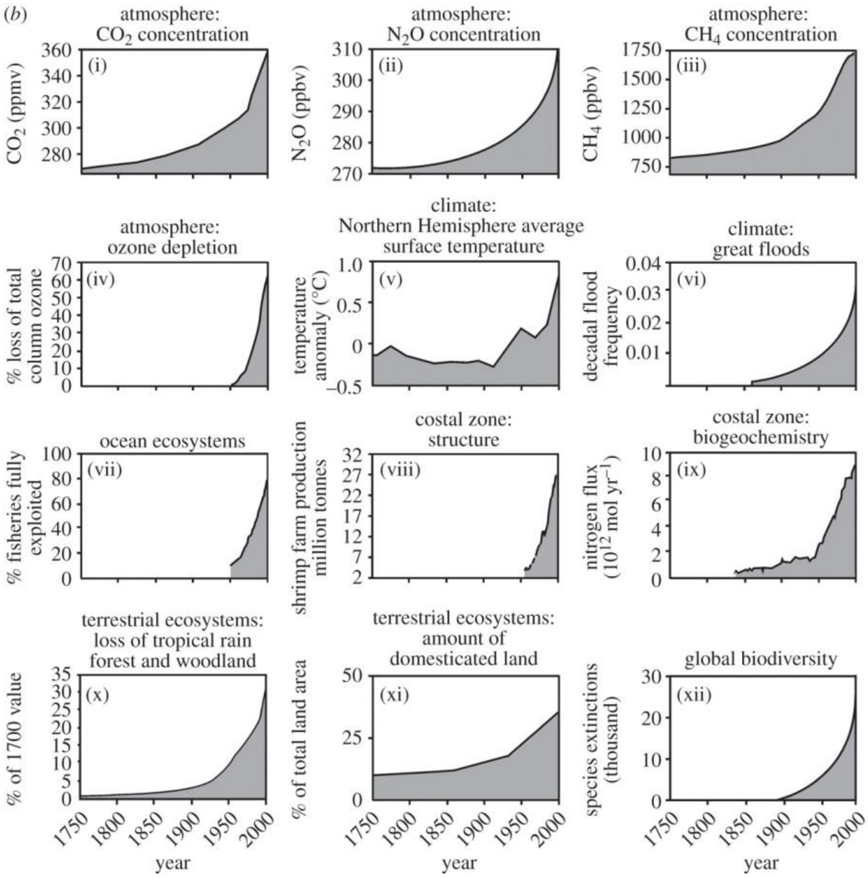

Figure 2: The effect of human change on the planet (IGBP, 2004, p.133)

The two figures (Figure 1 and 2) do not propose or suggest any direct causality from one graph to another. Rather, together, they offer a graphic illustration of accelerating human activity at a planetary scale and accelerating changes in geology and biology at a planetary scale. They point to the complexity of relations and the sheer number of potential stresses on the planet.

Linking planetary issues to education, Morgan (2016) suggests that, as a species, we have for over a century been operating under the influence of a ‘carboniferous capitalism’, with imagined unending economic development dependent on use of fossil fuels. Education has similarly, Morgan argues, been caught in an assumption of unending progress in the sense of being linked to offering increasing numbers of students access to the ‘good life’, a mind-set in which education is a consumer product with the promise of maximising an individual’s earning power. We could say that we have been caught in a *growth mind-set* of desiring unending progress within education (and social theory more generally) and, in the process, divorced our thinking from a connection to the limited natural resources of the world.

The recognition of the impact of collective human actions on a geological scale and the label ‘Anthropocene’ is a controversial one, in part due to controversy over what might be meant by ‘human’. There is a move within the social sciences to consider the non-human and to present the current time as ‘post-human’ (e.g., Hynes, 2016). Within such a context, the term ‘Anthropocene’ has been appropriated to signal the shift from hopes of ‘saving nature’ and ‘solving’ problems, to living with crises and problems as our new and permanent condition (Purdy, 2015). What would it mean to reconceptualise problems, crises, challenges, in other-than-human terms? The challenge of post-humanism in relation to mathematics education is taken up by de Frietas and Sinclair (2014) who cite their work within the movement of ‘new’ or ‘inclusive’ materialism. de Frietas and Sinclair draw attention back to the body and the material in the learning of mathematics and, in doing so, radicalise what we might think of as a ‘body’. From their viewpoint bodies are dynamic, dispersed, malleable, distributed and composed of material relations. Thinking, therefore, also moves away from its location in a singular brain, to take place within these dynamic and distributed relations involving others and the world. Humans can no longer be conceived as pre-existing subjects passively receiving experiences and actively representing the world in order to act, rather we are constantly involved in a process of becoming, including ‘becoming’ mathematics as we partake of the materiality of concepts and concepts partake of us in their own process of becoming.

Once we break the orthodoxy of assuming schools are constituted by collections of singular thinking subjects, to include material bodies in the post-humanist sense, it is a short step to extending the scope of this materiality beyond the classroom. The work of de Frietas and Sinclair (2014) is significant in acknowledging the body and the material; latent in their posthumanist approach is the sense that the relations that constitute the body and mathematics can extend to the whole planet. So, one move from considering the Anthropocene is a *de-centring of the subject* in the sense of moving away from a view of cognition as occurring inside the heads of individuals, to considering human relations in their widest, global sense. This is not to suggest such a move is new or un-acknowledged from other perspectives, e.g., enactivism (Reid and Mgombelo, 2015) or the work of Bateson (1972).

Hynes (2016), also from a post-human perspective, points to the importance of the aesthetic in allowing a way through the seeming dichotomy between championing specific values in an absolute manner and the opposite extreme of seeing all values as relative and equivalent. An absolute view of value assumes there is a transcendent morality, emanating from an all-knowing subject (typically male and white) such that the actions of others can be judged against a static framework. Relativism is simply the negation of this view and so comes from the same perspective in terms of locating morality in the actions of singular humans. Varela (1999) likewise proposes moving away from normative ethical standards to a perspective of engaging in the development of ‘ethical know-how’. Ethical behaviour is about on-going relationship not adherence to a set of values. To engage in the process of developing ethical know-how, Varela proposes three gestures of becoming aware: suspending, re-directing and letting-go (Varela and Scharmer, 2000). In moments of dilemma or choice, if we want to act out of awareness, we need to suspend habitual patterns, re-direct our attention towards the detail of the situation we are in and let-go of emotional attachments to past ways of seeing and acting.

Working with uncertainty and paradox are cited from a number of sources, as values needed for the future. Skovsmose refers to paradox when asked, in interview, about the future of critical mathematics education (Alro, et al., 2010, p.8); deFrietas and Sinclair (2014) explore paradox as a ‘pivotal [facet] of mathematical activity’ (p.43); Stinson and Bullock (2012) call for a ‘praxis of uncertainty’ in mathematics education. These are moves consistent with a recognition of the complexity of the systems human activity now influence. In a very real way we cannot know what to do and yet must still act. The concept of the Anthropocene pulls away from a focus on desired ‘ends’ for education (a shift noticeable in Barwell’s (2013b) work on climate change), in the face of the uncertainty of the future. This is not, however, an argument for the relativism of all actions and values, nor a reason for inaction in the face of global and societal crises. I see one challenge in the Anthropocene as the steering of a path between falling into the twin traps of relativism and inaction on the one hand and a normative and therefore alienating view of things that ‘should’ be done, on the other hand.

4. Teaching and learning mathematics

What does the discussion above mean for the teaching and learning of mathematics? Following Mendick (in press), who was inspired by Ahmed’s (2010) ‘happiness archive’, the next section is my own mathematics education archive, ‘structured through my own encounters’ (p.x), in thinking about teaching and learning mathematics in the Anthropocene. I consider three scenarios, chosen with a sense of what a sequence of activity might look like for a teacher or teacher educator wanting to engage in questions thrown up by the Anthropocene. The first section (4.1), informing ourselves, takes an example of exploring one question linked to sustainability. The second section (4.2) moves into the classroom, to consider how teaching might be planned or approached and the third section (4.3) asks how we might begin to think differently about student understanding of mathematics.

4.1 Informing ourselves

Part of any concern with global issues and challenges must involve informing ourselves and indeed some kind of critical statistical awareness is a part of desired skill sets of 21st Century learners (eg, Triling and Fadel, 2009). But what does this mean in practice? To help unpick the issues I offer below a narrative based on a discussion that took place in 2015 between three colleagues (myself, Piers and Peter), which was provoked by an article in a newspaper, linked to the question of green house gas emissions.

Piers emailed a link to a newspaper article written by George Monbiot, a well-known commentator in the UK on environmental issues. The article began as follows:

> The figures were so astounding that I refused to believe them. I found them buried in a footnote, and assumed at first that they must have been a misprint. So I checked the source, wrote to the person who first published them, and followed the citations. To my amazement, they appear to stand up.
>
> A kilogramme of beef protein reared on a British hill farm can generate the equivalent of 643kg of carbon dioxide. A kilogramme of lamb protein produced in the same place can generate 749kg. One kilo of protein from either source, in other words, causes more greenhouse gas emissions than a passenger flying from London to New York.
>
> This is the worst case, and the figure comes from a farm whose soils have a high carbon content. But the numbers uncovered by a wider study are hardly

reassuring: you could exchange your flight to New York for an average of 3kg of lamb protein from hill farms in England and Wales. You'd have to eat 300kg of soy protein to create the same impact. (Monbiot, 2015, para 1-3)

You might want to consider your own reaction to these paragraphs before reading the three messages Peter, Piers and I exchanged.

Peter:

I think George M is great and his articles generally hit home. But this doesn't make sense to me. A recent government select committee report http://www.eblex.org.uk/wp/wp-content/uploads/2013/05/Beef-and-Lamb-APPG-The-carbon-footprint-of-the-beef-cattle-and-sheep-sector.pdf quotes the CO_2 emissions for upland cattle at 15.7 kg CO_2 /kg liveweight and the equivalent for sheep at 13.6

Multiply these figures by 50% to account for the wastage from the carcass (I checked this on a couple of websites) and the figures are 24 kg CO_2/kg of beef and 20 kg CO_2/kg lamb approx. How does he get to 643 kg CO_2 for beef and 749 kg CO_2 for sheep?

I guess he is also talking about beef and sheep **protein** which is only about 27% of the piece of beef we eat. But even allowing for this you would only get to 89 kg CO_2 /kg beef protein and 74 kg CO_2 for lamb protein.

On the other hand the ETA says: *On every flight to New York and back, each traveller emits about 1.2t of CO_2, using Department for Transport figures. This compares to an average British personal total of 9.5t.* These are figures I recognize. Monbiot says a transatlantic flight is equivalent to 3 kg sheep protein. By my calculations above it should be 8 kg sheep protein (600 (the kg of CO_2 of a flight) divided by 74 (the kg CO_2 / kg of lamb protein)). And 8kg of sheep protein equals 29kg lamb, which should give you almost 200 meals at 150 grammes per person … 4 meals a week all year.

My figures sound a lot more reasonable … or have I got something horribly wrong? Needs a mathematician to check it out so I will copy in Alf

Alf:

Thanks for copying me in - fascinating. I can't fault your figures Peter.

I followed up on George Monbiot's sources, and the footnote he mentions refers to Nijdam et al. (2012), which I was able to access. In this article, to take Beef as an example, figures vary from 9 to 129 kgCO_2e/kg liveweight. The highest values (129) come from 'extensive' beef farming, as there is the least 'efficient' transfer of feed to meat (e.g., because the cows have to walk around). Included in these figures are any carbon-use linked to fertilisers, for example, and also methane emissions from the cattle. Welsh (extensive) beef was one the highest figures and they report that around 75% of the CO_2 figure came from 'enteric fermentation' which I think is a reference to methane emissions from the animals (I imagine that intensively farmed animals reach slaughter weight more quickly and so this percentage is a lot lower).

When the authors then compared protein, the figures jumped to 45-640 kg CO_2e/kg protein; this looks like a factor of 5 compared to liveweight, which is close to your assumption, Peter, that liveweight is 27% protein; their assumption seems to be 20%. And these are the figures quoted by Monbiot - again, the extensive use count for the highest figures. They are staggeringly high.

I looked up a later article talking about all this (Roos et al, 2013) that suggested: 'Beef production relying on grazing on natural and semi-natural grasslands can be very energy-efficient and cause little toxicity impact, since small amounts of energy and pesticides are needed for pasture management, but still cause large [Green House Gas] emissions due to CH_4 emissions from enteric fermentation.' (p.573).

From which I take it that carbon footprint isn't the last word on the issue (toxicity, biodiversity, etc, are also important). These authors were in favour of using carbon footprint figures to inform policy but also suggested that one danger is that the carbon footprint can mean little more than a measure of land use. Well, at least that's my tuppence-worth on what I could make out of the issue ...

Piers:

A typical 'serving' of meat contains about 22g protein – so, taking Monbiot's 1kg claim, about 45 servings is roughly equal to the CO_2 of one seat on a transatlantic flight - still pretty significant. For those of us that eat meat only a couple of times a week, that's still a return flight across the Atlantic each year. Interesting too that extensive farming - if you crunch the numbers - comes out as significantly worse (in terms of CO_2) than intensive, although I expect there are other benefits of extensive that might counteract this …

There is a paradoxical sounding conclusion we seem to reach in this exchange that extensive, hill farmed sheep cause more CO_2 emissions than animals raised intensively. Brand (2010) suggests, in what might be a similar inversion of typical thinking, that city living is the most sustainable way of life on the planet. The density of people and economies of scale mean that from providing water, to transport, to dealing with waste, when calculated per person, emissions are lower for cities than rural living. The short life and lack of movement of intensively reared cattle, while raising its own concerns both ethically and in terms of toxicity, appears to mean meat from this source has a smaller carbon footprint than meat from cattle that are less intensively reared.

However, from the article and email exchange, we have arrived at three different possible figures. Monbiot claims, depending on the source, either 1kg or 3kg of sheep protein causes the CO_2 emissions of a transatlantic flight. Peter's calculations, using government figures, suggest 8kg. Taking Piers' 22g of protein a serving, this is the difference between a single flight equating to 45 servings (less than once a week for a year), 136 servings (2 or 3 times a week) or 364 servings (eating lamb each day for a year).

Peter had initially been concerned that his own calculations (8kg instead of Monbiot's initial claim of 1kg) were so far apart that there must have been some error (on his or

Monbiot's part). In the original Monbiot article, I wonder how many readers pick up the distinction between a kg of meat protein and a kg of liveweight, just from those first three paragraphs quoted above. I know for myself, I was imagining the 1kg being talked about as the size of a leg of lamb I might see in the supermarket, i.e., potentially one family meal. There is nothing ambiguous in the article, when looked at carefully, except perhaps that when we talk about meat by weight we rarely refer to protein. In order to make the conversion from protein to weight of meat (a factor of 4 or 5) and then from weight of meat to liveweight (a factor of 2, according to Peter's research) requires considerable background knowledge.

The conversation narrated above is, perhaps, an example of critical engagement with the media as citizens. It has been interesting to list some of what was involved in Peter's analysis (independent of any technical mathematical skills): an initial sense of CO_2 figures in order to question them; making sense of the units kg CO_2/kg liveweight; knowing that there would be government figures for CO_2 emissions from cattle and sheep and how to access them; researching the proportion of protein in a certain liveweight; knowing the CO_2 emission figures for a transatlantic flight; knowing how much protein might go into a typical meal. There are then requirements such as good internet access and the time to engage in researching around figures (in a postscript to his email, Peter added 'can you tell I am on holiday').

Unsurprisingly, there is uncertainty in our conclusions. The figures (for kg protein equivalence to a flight) range by a factor of eight and there is the paradoxical sounding conclusion about the high CO_2 emissions on extensive, hill farms. As Piers comments, there are surely benefits to this kind of extensive farming. In terms of productivity of land, there is perhaps little other use some of this land could be put to. In terms of toxicity and biodiversity there are likely benefits of extensive farming over intensive. How is it possible then, as a citizen and particularly as a teacher, to begin balancing these agenda and how can we possibly inform ourselves in the manner Peter is informed, about the kind of range of issues suggested by Figures 1 and 2, in order to critically question in the first place?

It might be possible to react to the ambiguity, complexity and uncertainty involved in thinking through the effects of our actions, with a sense of fatalism or disillusionment at ever having a sense of what we 'should' do. If it is that difficult to figure out something relatively simple (the carbon emissions linked to meat consumption) how can we go about thinking about raising such issues in a classroom? And yet, reflecting on the questions provoked by the Anthropocene, it is this state of ambiguity and uncertainty that is closely linked to the kind of thinking it is suggested we need to cultivate. If we believe we need to develop an ethics of uncertainty, part of this must involve accepting that there is no single answer to whether, say, intensive or extensively farmed meat is the more ethical choice. The desire to have an unambiguous resolution and have certainty in how we act comes from a position of setting ourselves apart from the world. It would only be if the world followed determined and relatively simple routines that we might be able to untangle cause and effect to the point of knowing the consequences of what we do. We can never know the effects of our actions and grappling with the complexity of the environmental consequences of different choices is a fairly quick way of coming face to face with this unknowing.

At the same time, through engaging in the process of making sense, there is a raising of awareness. While we cannot know precisely the equivalence between meat eating and transatlantic flying, nor the variety of impacts of intensive compared to extensive farming, it seems pretty clear both flying and eating meat cause significant emissions. Through sharing what we notice, we can come to awareness of new relations and links, without needing to find resolution. In the next section, I continue this line of thinking into considerations of approaches to classroom teaching.

4.2 Approaches to teaching

The exchanges above, about CO_2 emissions from various sources, offers potential for modeling and engaging in technical or reflective knowing (Skovsmose, 1994) or, alternatively, any of Renart's (2011) three approaches (accommodation, reformation, transformation). As a teacher, how might I begin to decide what to offer in a classroom related to an issue, such as this one, that I have researched and become engaged in personally?

In discussions of different ways of learning or knowing mathematics there is often an implied hierarchy. Skovsmose's (1994) distinctions of mathematical, technical and reflective knowing has a sense that the reflective form of knowing is the highest. Similarly, Renart's (2011) distinctions between assimilation, reformation and transformation, even in the labels, carry a sense of transformation being the ultimate 'end'. The thinking implied by the Anthropocene wards against the 'growth' mindset that can be implied by such categories. In an always already compromised world, utopian visions of schooling can become self-defeating if they lead to dissatisfaction and disillusionment through their inevitable frustration.

Almost thirty years ago, Tahta (1988) raised a challenge for mathematics teacher educators, linked to a study by Desforges and Cockburn (1987), on the gap between the practice of new teachers and the aspirations of their teacher-trainers. Tahta suggested 'the teachers' job is more constrained, and more complex, than sometimes seems to be assumed by those who advise them how to teach mathematics' (1988, p.13), a statement that seems as relevant today as when it was written. The original study suggested:

> The teachers in our study seemed very conscious of the amount of material they had to cover ... When the authors of the Cockcroft Report exhort teachers to follow issue *x* or question *y* as these are spontaneously raised, they perhaps do not quite know what they are asking for. (Desforges and Cockburn, 1987, p131)

The reference to the Cockcroft report (a UK publication in 1982 on how mathematics teachers should teach) might now be replaced, within a context of an interest in sustainability, with notions of transformative learning or reflective knowing – if we make calls for changes in teaching, do we know what we are asking for?

In writings about the Anthropocene, there is an important recognition that we cannot, from here, return to any kind of unblemished state of nature (Purdy, 2015). Human activities are so intertwined with geological processes that there is no natural state to which return is possible and it is not even helpful to pretend otherwise. There is an analogy to be made that, perhaps, there is no utopian vision of mathematics education to which we can return or aspire. There have been recent suggestions (Pais, 2013) questioning the near-ubiquitous assumption of a use-value for mathematics and pointing to the impossibility of a mathematics education that can ‘empower’, while we are in the grip of what seems to be a ‘necessary’ (p.30) condition for the subject of mathematics in the accreditation and hence (social) selection of students. This does not, however, need to be a pessimistic conclusion. The provocation of the Anthropocene is to provide an injection of energy and interest in a recognition of the fallen and compromised state of the globe. Similarly it need not be a cause of disappointment or inaction to recognise that dreams of, for example, ‘mathematics for all’ will never be realised. Rather than look to an ideal image of mathematics teaching as an aspiration (e.g., a transformative approach) the always-already blighted and ruined thinking in the Anthropocene might suggest taking seriously teachers’ concerns about, say, the material to cover in the curriculum.

If we do take seriously teachers’ concerns (seemingly expressed worldwide) about the lack of time for the study of mathematics, there is an injunction to focus on the efficiency (in terms of time) of teaching and learning mathematics. Indeed, Tahta (1989) suggested that, as a field, we had paid too little attention to the actual mechanisms by which students gain facility with symbols. If lack of time is indeed a significant factor limiting in teachers addressing wider issues such as problem-solving skills or real-world applications of mathematics or any considerations linked to sustainability and the Anthropocene, then finding ways to get students technical fluent with symbols in a shorter amount of time that is taken currently, could allow for this other work to take place. For example, reflecting on the work of Gattegno and Davydov (see Coles, in press), who both developed curricula that have been used in highly effective manners (Dougherty, 2008; Gattegno, 1974), suggests a commonality in the way that symbols are made to represent actions and the power that this can entail.

As a teacher committed to working on how mathematics can contribute to thinking relevant to the Anthropocene, in certain contexts a focus on mathematical or technical knowing and ensuring my students can gain access to higher education is surely an entirely appropriate aim. Greg Doran, the director of the Royal Shakespeare Company (RSC) in the UK, in this 400th anniversary of Shakespeare’s death (1616), was invited to give ‘The Richard Dimbleby Lecture’ in 2016. In this lecture, he described the engagement of one school in the RSC’s outreach programme and the way in which teachers and students had appeared to become inspired by the study of Shakespeare and the opportunity to work with professional actors. One anecdote from this school was a parent describing how they had decided they were going to vote, for the first time, in the next election. Seemingly, engaging in the study of Shakespeare, in its own terms, had led to a shift in how this parent viewed themselves as a citizen. And the same is surely possible in mathematics. Who is to say that a focus on mathematical knowing (in Skovsmose’s sense) offered with creativity and sensitivity, would not lead to just as much action or critical awareness (e.g., in terms of sustainability) as any other approach? Who is to say that a classroom focus on mathematical thinking in its own terms (which might not register on Renert’s (2011) scale), if it allowed students to gain a sense of their own power and agency over the subject, might not also allow students to reflect on their own actions and potential role in relation to global challenges?

Learning to teach entails the development of habits. The job is simply too complex for each decision to be contemplated consciously, and this is one of the impossibilities of becoming a teacher, that at the start of the process typically few habits are effective in the classroom. New behaviours and effective actions need to become automatic to allow attention to be given, for example, to what students say. Rather than exhort teachers to move up any imagined scale or continuum towards incorporating sustainability in lessons, a more productive approach might be to support a process of opening up habits and patterns to question and a deepening of awareness.

The importance of openness to change points towards the notion of ethical know-how (Varela, 1999) and the suggestion of three gestures of becoming aware: suspending, re-directing, letting-go (Varela and Scharmer, 2000). One of the provocations of the Anthropocene is precisely in the possibility of placing my teaching within the context of planetary processes in order to provoke a suspending of habitual patterns of behaviour. Without any injunction about how teaching should then take account of such a context, the uncertainty of what to do is productive to the extent it allows suspension and a re-direction of attention towards alternative way of acting in the classroom. Whether introducing a topic on climate change or the quadratic formula, as a teacher I am constrained by the routines and expectation established with any class I have taught while still, of course, being in a position to innovate and provoke change in those expectations. No matter what my approach to teaching, there is nothing to stop my working on *how* I interact with my students. An aesthetic move, for example, might suggest times of listening to students without evaluation (Davis, 1996; Coles, 2001). In doing this there is an implicit recognition that views are not fixed and that through participating together in an event we involve ourselves in a process of becoming.

The next section completes the imagined journey, in this paper, from considering an issue, to considering approaches to teaching, to considering the learning of students.

4.3 'Understanding' and 'not-understanding' mathematics

The Anthropocene, interpreted in a post-human manner, prompts thinking in non-human, non-subjective ways. In terms of learning mathematics, an almost universal assumption appears to be that the appropriate aim for education is for students to *understand* the subject (how could it be otherwise?); yet, how might we think about this in non-human terms?

There is a vast literature on understanding in mathematics (Skemp, 1976/2006; Sfard 1994; Pirie, 1989; Rittle-Johnson and Schneider 2015; to name just a few). It is not my intention to review this literature in any systematic manner, but there do appear to be some common themes. For example, Sfard (1994) developed the notion that the learning of mathematics requires the 'reification' of processes and procedures. Her suggestion was that we attain conceptual knowledge through a developmental process that starts with these processes and procedures. We arrive at metaphors or 'figurative projections from the world' which serve as the 'basis for understanding' (p.52). Pirie and Kieran (1994) come to a remarkably concurrent notion to Sfard's of how we develop understanding, through starting with the 'fundamentally metaphoric' (p.43), although they take a quite different view of what understanding *is*. For Pirie and Kieran, understanding is a dynamic and complex system that moves between noticing and understanding images and properties.

The emphasis on the metaphoric is something echoed much later in the work of Lakoff and Nunez, (2000) and their depiction of the bodily basis of understanding mathematics. Across all these authors, then, metaphorical and primary processes are replaced and super-ceded by imagined or virtual processes which in turn make way for more structural, formal and conceptual knowing. The accepted vision here seems to be one of basing mathematical knowledge on metaphors of physical actions, so that students will attain conceptual knowledge.

Through all these studies, understanding is seen as a subjective quality, characteristic of particular individuals who are assumed to possess intelligence and diligence and for whom the benefit of understanding comes, perhaps, as a morally just consequence of their virtue (and normality). What alternative narratives might there be to this individualistic and (implicitly) moralistic viewpoint of development towards the end point of conceptual knowing?

A post-human view of understanding might begin by thinking about what happens when understanding is *not* seen to be around. While there are countless articles and books on understanding, its shadow 'not understanding' is given relatively little attention. Yet, one of the problems of a focus on understanding mathematical concepts is that, to quote again from Dick Tahta (1989):

> [t]his, then, inevitably leads to metaphors of ownership and control: obtaining the meaning, having the understanding, getting the concept. And, consequently, of course, there will be the mathematical *descaminados*, the shirtless who have not understood, who never get the concept … I always have some concept of what we may both be considering. I will certainly never have yours (p.2)

Those groups (often the 'other' in society, the marginalised) who do not understand are perhaps assumed to lack intelligence and maybe diligence. Their lack of understanding, stemming from their singular characteristics, is a justification for exclusion, for example from prosperity or opportunity.

Understanding a concept suggests an individual making free choices who is able to demonstrate their competence. Therefore 'not-understanding' also must imply a freedom to act. While of course context and the social and societal and language and the body may now be brought into discussions of student understanding, the assumption of free action continues to suggest a human divorced from nature and able to get in touch, or not, with something outside nature. 'Not understanding' is a deficit and a lack of connection. A post-human re-casting of 'not understanding' might start from here: is it possible to approach 'not understanding' differently, not as a deficit?

Hynes (2016) re-frames 'indifference' as an 'ambiguous process' (p.33) and something similar seems productive in thinking about 'not understanding'. If we consider some of the values suggested earlier in this article as consistent with thinking in the Anthropocene (uncertainty, paradox) these are *precisely* a description of what it is like to be in a state of 'not understanding'. It is when a student is *not* sure of what they are doing that they have the potential to inhabit all of the behaviours held up as necessary for acting differently in relation to global challenges.

A vignette may help to explore the potential for ‘not understanding’. The incident below took place at my home and was written up shortly afterwards.

> My daughter (age 7) was doing her mathematics homework next to me. She had a worksheet with questions about bar charts and one question involved adding up the totals of all the bars, which were: 140, 190, 130, 210, 180. She initially wrote out the five numbers in a column, but without precisely aligning them and arrived at the total 697. I said I didn’t think that was correct and wrote out the five numbers again, this time aligning the digits in three columns. She said “Oh, yes, I know what to do” and starting adding up the ‘hundreds’ column. She arrived at the answer 6270. I made a comment about the ‘2’ needing to join the ‘6’. She wrote an ‘8’ underneath the 6 in her answer but then wanted to start again and she wrote out the numbers once more, aligning them correctly. She wrote an ‘8’ as the total for the hundreds and then, apparently in frustration, said “Oh, I can’t do it”, left the desk where she was working and lay down on the floor. I told her I thought she was very close and could come back to try again, but she did not acknowledge the request. After a short time, I asked her if she would like to do the sum on a calculator on my computer; she immediately got up and started to work with the calculator, typing in the numbers to get the correct answer.

It is evident in this story that my daughter has some confusion about a process she has been taught for adding numbers. Without wanting to get into discussion about what she does or does not understand, it seems clear that there is ambiguity and uncertainty for her, in relation to the mathematical task she has been asked to perform. However, her initial answer, 697, did not appear her to strike her as incorrect and I suspect without my presence this is what she would have written down so in some sense her uncertainty is provoked by me. In what ways can this provoked-uncertainty be seen as productive, given that it seemed at one point to lead to frustration and a disengagement from the task?

In reflecting on the incident, I recognise I missed an opportunity for a (meta)-comment after what occurred. I might have said something like: ‘if you feel you can’t do something there is often a tool that will make it possible’. The state of not-understanding would be necessary, in this instance, for the making of a connection that was potentially useful in terms of supporting effective action in future. However, on this reading, it is only in the resolution of uncertainty that the state of not-understanding becomes productive. The challenge of the Anthropocene is to think through how being in a place of non-understanding might become productive in itself.

In another scenario, I might have (meta)-commented to my daughter, when she was on the floor that she seemed to be stuck/frustrated and tried to engage in a discussion about strategies of what to do next. The Open University, in the UK, many years ago produced a poster to accompany some of their mathematics education courses that began ‘STUCK? Sit back and enjoy it’ before then listing some possible strategies of what to do next. I suspect my daughter may not have wanted to engage in such a discussion at that moment, although the readiness with which she moved to the calculator might suggest she was committed to completing the work. How might we work, as teachers, so that not-knowing is celebrated as a way of being/becoming?

Perhaps one of the anxieties about not-understanding is linked to the individual focus of mathematical learning in school. As an alternative to a focus on individual understanding, Tahta (1989) called for a 'communal mathematics'. Instead of a concern about what students do or do not understand, I might pay attention to what students, collectively, can *do*. Collective and communal activity pulls away from the individual subject as the prime concern. One example of a way of working is communal chanting. Oral recitation is a practice often seen in African countries and equally often interpreted as an 'unenlightened' teaching strategy. And yet, there are possibilities in asking for communal response, for students to be carried along by the work of others to surprise themselves in what they are able to do and for the teacher to gain and work with immediate feedback from the whole group. Gattegno (1974) developed a number chart that can be used powerfully in a communal manner (e.g., see Coles, 2014). The focus of such work is unlikely to be in the realm of reflective knowing and yet may offer opportunities for students to experience the power of their own learning in the sense of being able to do things they could not do have done alone, and in a short space of time.

5. Concluding thoughts

This article aimed to consider issues relevant to mathematics education in the light of a recognition of human influence on planetary processes and a sense we have moved into a new geological era, the Anthropocene. A post-human reading of the concept of the Anthropocene moves away from a sense of 'growth' (Morgan, 2016) and the growth mind-set that sets up specific and defined 'ends' for education. Instead, attention is brought back to the present-moment of interaction and the development of ethical know-how, the cultivation of uncertainty and paradox as a productive and celebrated state.

This thinking was used to reflect on an 'archive' of encounters: through informing myself about an issue, to planning to teach and finally thinking about the learning of students. The complexity of informing ourselves about issues around sustainability was apparent in a discussion of something as seemingly straightforward as the CO_2 emissions associated with meat production. The impossibility of fully informed action was apparent and yet it is possible to value the potential productivity of a state of uncertainty and unknowingness. It is through coming face to face with our incapacity to know what is the ethical course of action, that we can recognise we are caught in paradox and turn focus to our awareness of present-moment relationships and interactions.

In terms of planning to teach mathematics, a move away from focusing on desired 'ends' throws into question hierarchies such as seeing teaching for sustainability in terms of: accommodation, reformation, transformation (Renart, 2011). Such categorisations can become useful tools, not to judge schools or teachers and notice deficiencies, but in order to support teachers' own judgments about what is appropriate for the students they teach (now). It may be tempting to see the end of any list of categories as the 'best' but there may be good reasons, in particular contexts, for a focus on passing an examination, for example, in providing access to further opportunities. Deep understanding of mathematics is often held up as a desired end for mathematics education. But it is surely also important to recognise that, in doing mathematics, it is vitally important that I can use symbols or perform algorithms in a completely unthinking, unmarked manner – precisely so that my attention can be on any further purposes or uses for mathematics, if that is something on which I want to focus.

Finally, in considering the learning of students, there was consideration of what can be an unvalued state in the classroom, of not understanding. In the moment of not-understanding mathematics we may well be caught in a paradox of assumptions, almost certainly faced with uncertainty and perhaps face-to-face with a sense of the uncanniness of those others not in a similar state. What might it take, as a teacher, to develop a classroom environment in which not-knowing was celebrated? And, how might I move away from a focus on individual understanding in the first place, perhaps to develop a communal mathematics?

One of the paradoxes of teaching is that while we are preparing students for a future world that is uncertain and unknowable, our teaching must take place now. The thinking reviewed and used here, linked to the Anthropocene, suggests that in the face of such uncertainty, we bring focus to the present moment of our interactions. The idea of the Anthropocene, in itself, can provide a mechanism for disrupting patterns of interaction and bringing awareness to my on-going ethical relationships with everything around me.

References

Ahmed, S. (2010). *The promise of happiness*. Durham: Duke University Press.

Alrø, H. Ravn, O. and Valero, P. (2010) Inter-viewing critical mathematics education. In, H. Alrø, O. Ravn, and P. Valero (eds.), *Critical Mathematics Education: Past, Present and Future*, Sense Publishers: Rotterdam, 1–9

Barbosa, J.C. (2006). Mathematical Modelling in classroom: a critical and discursive perspective. *Zentralblatt für Didaktik der Mathematik,* 38(3), 293-301.

Barwell, R. (2013a). *The role of mathematics in shaping our world*. In A. Coles, R. Barwell, T. Cotton, J. Winter, & L. Brown (Eds.), *Teaching Secondary Mathematics as if the Planet Matters*. Routledge: London & New York, pp. 3-15.

Barwell, R. (2013b). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education, 15*(1), 1-16.

Bateson, G. (1972). *Steps to an ecology of mind*. Chicago: University of Chicago Press, 2000.

Brand, S. (2010). *Whole earth discipline: why dense cities, nuclear power, transgenic crops, restored wetlands, radical science and geoengineering are necessary*. London: Atlantic Books.

Coles, A. (2001). Listening: A Case Study of Teacher Change. In M. van den Heuvwel-Panhuizen (Ed.), *Proceedings of the twenty-fifth annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 281-288). Utrecht, Netherlands: PME 25.

Coles, A. (2013). *Biodiversity*. In A. Coles, R. Barwell, T. Cotton, J. Winter, & L. Brown (Eds.), *Teaching Secondary Mathematics as if the Planet Matters*. Routledge: London & New York, pp. 16-23.

Coles, A. (2014). Transitional devices. *For the learning of mathematics, 34*(2), 24-30

Coles, A. (in press). A relational view of mathematical concepts. In E. deFrietas, N. Sinclair, A. Coles (Eds.) *What is a mathematical concept?* Cambridge: Cambridge University Press.
Crutzen P. J., Stoermer E. F. (2000) The Anthropocene. *Global Change Newsl.* **41**:17–18.
deFreitas, E., and Sinclair, N. (2014). *Mathematics and the body: material entanglemenats in the classroom.* Cambridge University Press: Cambridge
Davis, B. (1996). *Teaching Mathematics: Toward a Sound Alternative*. New York: Garland Publishing Inc.
Desforges, C., and Cockburn, A. (1987). Understanding the mathematics teacher, Falmer Press,
Dougherty, B. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahweh, NJ: Erlbaum.
Finley, F. (2014). The Anthropocene and the Framework for K–12 Science Education. *Future Earth-Advancing Civic Understanding of the Anthropocene*, 9-17.
Gattegno, C. (1974). *The common sense of teaching mathematics*. New York: Educational Solutions Worldwide Inc. (reprinted 2010).
Hynes, M. (2015). Indifferent by nature: A post-humanist reframing of the problem of indifference. *Environment and Planning A*, 0308518X15594621.
IGBP (2004). *Global change and the Earth system: a system under pressure*. Springer: New York. Available at: http://www.igbp.net/publications/igbpbookseries/igbpbookseries/globalchangeandtheearthsystem2004.5.1b8ae20512db692f2a680007462.html (accessed 11th Feb 2016)
Kaiser, G., and Sriraman, B. (2006) A global survey of international perspectives on modelling in mathematics education, *ZDM 38*(3): 302-310**.**
Lakoff, G., & Nunez, R. (2000). *Where mathematics comes from: how the embodied mind brings mathematics into being*. New York: Basic Books.
Mendick, H. (in press). Queering mathematical concepts. In E. deFrietas, N. Sinclair, A. Coles (Eds.) *What is a mathematical concept?* Cambridge: Cambridge University Press.
Monbiot, G. (2015) Warning: your festive meal could be more damaging than a long-haul flight. The Guardian, 22nd December 2015. Available at: http://www.theguardian.com/commentisfree/2015/dec/22/festive-christmas-meal-long-haul-flight-meats-damaging-planet (accessed 17th April 2016)
Morgan, J. (2016). Post-carbon future for Geography education. In M. Robertson and E. Po-Keung (Eds) *Everyday Knowledge, Education and Sustainable Futures*. New York: Springer.
Nijdam, D., Rood, T. and Westhoek, H. (2012), 'The Price of Protein: Review of Land Use and Carbon Footprints from Life Cycle Assessments of Animal Food Products and their Substitutes', *Food Policy*, *37*: 760–70.
Pais, A. (2013). An ideology critique of the use-value of mathematics. *Educational studies in mathematics*, *84*: 15-34.
Pirie, S. (1989). A recursive theory of mathematical understanding. *For the learning of mathematics*, *9*(3), 7-11.
Purdy, J. (2015). *After Nature: A Politics for the Anthropocene*. Harvard University Press.
Reid, D., & Mgombelo, J. (2015). Soots and key concepts in enactivist theory and methodology. *ZDM, The International Journal on Mathematics Education, 47*,

171–183.
Rittle-Johnson, B., Schneider, M. (2015). Developing Conceptual and Procedural Knowledge of Mathematic. In R. Cohen Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition.* Oxford: Oxford University Press.
Roos, E., Sunberg, C., Tidaker, P., Strid, I., Hansson, P-A. (2013). Can carbon footprint serve as an indicator of the environmental impact of meat production? *Ecological Indicators, 24,* 573–581.
Sfard, A. (1994). Reification as the birth of metaphor. *For the learning of mathematics, 14*(1), 44-49.
Skemp, R. (1976/2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*, 12(2), 88–95. Originally published in *Mathematics Teaching*.
Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.
Stinson, D., and Bullock, E. (2012). Critical postmodern theory in mathematics education research: a praxis of uncertainty. *Educational Studies in Mathematics, 80*: 41-55.
Tahta, D. (1989). *Take care of the symbols*. Unpublished paper.
Trilling, B. and Fadel, C. (2009). *21st century skills: Learning for life in our times*. San Francisco, CA: John Wiley & Sons.
Varela, F. and Scharmer, O. (2000). *Three Gestures of Becoming Aware. Conversation with Francisco Varela January 12, 2000, Paris*. Available at: https://www.presencing.com/sites/default/files/page-files/Varela-2000.pdf (accessed 11th Feb 2016).
Wigner, E. P. (1960). "The unreasonable effectiveness of mathematics in the natural sciences. Richard Courant lecture in mathematical sciences delivered at New York University, May 11, 1959". *Communications on Pure and Applied Mathematics 13*: 1–14.